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Foreword

Academic success can be measured in many different ways, and I often tell my students that scoring high marks in exams is only one of the rewards from diligent study. The true measures of academic success should be the enjoyment of learning and the sense of accomplishment students get when that light flicks on in their head and they think to themselves, “So that’s why!” The inception of NTK’s study guides and publications is based on the simple goal of making students’ learning process more enjoyable and less complicated, and to deliver positive results from students’ efforts.

The International Baccalaureate (IB) Middle Years Programme (MYP) is designed to help students recognize the connection between what they learn in the classroom and the world around them, to tie the various subject areas together, and eventually to help students “see knowledge as an interrelated, coherent whole”. NTK’s IB MYP Mathematics Level 5 Study Guide helps students review and solidify concepts in preparation for the more challenging IB Diploma Mathematics programme. In order to achieve high scores in IB Mathematics, it is essential for students to first have a solid understanding of the mechanisms involved before they can confidently establish exam strategies. We believe that this study guide will help students do just that.

NTK’s study guides, courses and educational services are designed to help prepare students for exam success as they continue to pursue secondary and college education. As a leading educational service provider in Southeast Asia for more than a decade, NTK has helped thousands of students reach their academic goals. Whether they are in primary, secondary, or post-graduate studies, our students have benefited greatly from our specialized academic programs and expertise in all major international curricula and exams.

As you continue on your studies, I wish you every success and most importantly, I hope you enjoy the learning process as well.

T.K. Ng
Founder and Managing Director
NTK Academic Group

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Chapter 1
Sequences and Series

Section 1.1
Arithmetic sequences

An arithmetic sequence is a sequence in which the difference between consecutive terms is constant. We can obtain the next term by adding the same number each time.

Consider the sequence

$\{2,\ 6,\ 10,\ 14,\ 18, ...\}$

The difference between consecutive terms is 4.

The difference between consecutive terms does not need to be positive.

Consider the sequence

$\{25, 16, 7, -2, -11, ...\}$

The difference between consecutive terms is –9.

The n^{th} term of the arithmetic sequence is given by:

$$u_n = u_1 + (n-1)d$$

u_n is the n^{th} term of the arithmetic sequence

u_1 is the first term of the arithmetic sequence

d is the common difference $(= u_n - u_{n-1})$

Arithmetic sequences are also referred to as arithmetic progressions.

Example

Given the arithmetic sequence $\{4, 7, 10, 13, ...\}$, find the 99th term.

Solution:

We need to find d and u_1 first:

$u_1 = 4$, since 4 is the first term.

For d, take any pair of consecutive terms, such as the first and second terms.

$$\begin{aligned} d &= u_n - u_{n-1} \\ &= u_2 - u_1 \\ &= 7 - 4 \\ &= 3 \end{aligned}$$

Using the formula,

$$\begin{aligned} u_{99} &= u_1 + (99-1)d \\ &= 4 + (99-1)3 \\ &= 298 \end{aligned}$$

Example

Given the 3rd term and 7th term of an arithmetic sequence as –4 and 16 respectively, find the 20th term.

Solution:

Again, we need to find d and u_1 first.

Using the formula,

$$\begin{aligned} u_3 &= u_1 + (3-1)d \\ u_7 &= u_1 + (7-1)d \end{aligned} \Rightarrow \begin{aligned} -4 &= u_1 + 2d \\ 16 &= u_1 + 6d \end{aligned}$$

Then we use the elimination method by subtracting the second equation from the first equation:

$$\begin{aligned} -20 &= -4d \\ d &= 5 \end{aligned}$$

Put $d = 5$ into either one of the equations. We will use the first equation in this case.

$$\begin{aligned} -4 &= u_1 + 2(5) \\ u_1 &= -14 \end{aligned}$$

Therefore, the 20th term is:

$$\begin{aligned} u_{20} &= u_1 + (20-1)d \\ &= -14 + 19(5) \\ &= 81 \end{aligned}$$